surface $(p(r)=0)$, we arrive at a problem of determining the values of the parameter λ for which non-trivial solutions exist for the homogeneous Eq.((2.4) for $F(x)=0$). The minimum value of the parameter $\varepsilon=1-\lambda$, for which this equation has a non-zero solution is the critical deformation for which axisymmetric and symmetric buckling of the layer material relative to the $z=0$ plane occurs near the crack, i.e., opening of the crack occurs because of elastic instability. Below we give values of the critical deformation ε found by a numerical solution of the equations mentioned for a number of values of the relative half-thickness of the layer h_0

h_0	0.1	0.3	0.5	0.7	1
$\varepsilon\cdot 10^3$	7.22	50.9	111	168	228.

The author is grateful to V.A. Eremeyev and M.I. Karyakin for assistance in performing the numerical calculations.

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Translated by M.D.F.

PMM U.S.S.R.,Vol.52,No.2,pp.260-261,1988
Printed in Great Britain
0021-8928/88 $10.00+0.00

GRAVITATIONAL ACCELERATION IN MINKOWSKI SPACE* **

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The connections between models of the physical phenomenon of gravitation and geometrical models of space and time have been described previously /1-4/. As a result, an analysis was obtained of the macroscopic nature of gravitational interactions in the framework of four-dimensional pseudo-Riemannian spaces and three-dimensional Euclidean spaces in which there is Newtonian universal absolute time.

The corresponding theory is developed below for families K consisting of world lines associated with the free motion of individualized points that correspond to particles with a constant rest mass.

In the generalized coordinate system $\xi^1, \xi^2, \xi^3, \xi^4$ we find for individual points of the family K that $\xi^\alpha = \text{const}\ (\alpha=1,2,3)$ and ξ^4 is the time coordinate, changing along the world lines of K.

Generally speaking, for any family of associated world lines K, not necessarily for free motions of material particles in a pseudo-Riemannian space, we can introduce an associated canonical coordinate system $\xi^1, \xi^2, \xi^3, \tau$ where the metric has the following form at every point:

$$ds^2 = c^2 d\tau^2 + 2g_{\alpha 4} d\xi^\alpha d\tau + g_{\alpha\beta} d\xi^\alpha d\xi^\beta \quad (\alpha, \beta = 1, 2, 3)$$

where the coordinate τ coincides with the proper global time on the world lines of K, and the components of the acceleration on the world lines are given by the formulae

$$g_{\alpha 4} = u_\alpha, \quad a_\alpha = \frac{\partial u_\alpha}{\partial \tau} = \partial g_{\alpha 4}(\xi^\alpha, \tau)/\partial\tau$$

where u_α is the covariant component of the four-dimensional velocity vector $\mathbf{u}$, directed along the tangent at each point of the corresponding world line K, and the contravariant components

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*Prikl.Matem.Mekhan.,52,2,331-332,1988

**Paper read at the International Conference on "Modern Mathematical Problems of Mechanics and its Applications". Moscow, November 1987. Appendix to the paper "On the nature of time, space, and gravitation" (PMM, 51, 6, 1987).

of the vector $\mathbf{u}$ are equal to 0, 0, 0, 1.

As is well-known, the field of corresponding absolute acceleration vectors on the world lines of K with the generalized coordinates $\xi^\alpha = \text{const}$ is obtained in a local reference system by differentiating the three-dimensional velocity $\mathbf{v}$ with respect to the observer's time t along the lines of K or by differentiating the four-dimensional velocity $\mathbf{u}$ with regard to the proper global time τ.

We can determine the acceleration vectors by applying the local orthonormal tetrad bases $\mathfrak{z}_\alpha, \mathfrak{z}_4 = \mathbf{u}$ at each point of the family K of world lines on differentiating (with respect to t or τ, respectively) for constant $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^3$ and variable $\mathfrak{z}_4 = \mathbf{u}$ taking account of the equalities $\pm\mathfrak{z}_\alpha = \mathfrak{z}^\alpha$, which hold in locally-inertial orthonormal tetrads.

In these tetrads in local Newtonian absolute spaces we have the following for the absolute accelerations:

$$\mathbf{a} = \partial\mathbf{v}(\xi^\alpha, t)/\partial t = (\partial v_\alpha/\partial t)\,\mathfrak{z}^\alpha$$

and in a Riemannian space, specifically, a Minkowski space, we have

$$\mathbf{a}^* = \partial\mathbf{u}(\xi^\alpha, \tau)/\partial\tau = (\partial\mathbf{u}(\xi^\alpha, \tau(\xi^\alpha, t))/\partial t)\,\partial t/\partial\tau$$

where, on the basis of the Lorentz transformation, the following equalities hold in an associated canonical reference system:

$$d\tau = dt\sqrt{1 - \frac{v^2(\xi^\alpha, t)}{c^2}}, \quad u_\beta = \frac{v_\beta}{\sqrt{1 - v^2/c^2}}.$$

Hence, the following formulae follow:

$$\mathbf{a}^* = a_\alpha{}^*\mathfrak{z}^\alpha = \frac{\partial u_\beta}{\partial\tau}\mathfrak{z}^\beta = \left\{\frac{\partial}{\partial t}\left[\frac{v_\beta}{\sqrt{1 - v^2/c^2}}\right]\right\}\frac{\mathfrak{z}^\beta}{\sqrt{1 - v^2/c^2}}, \quad \mathbf{a} = a_\alpha\mathfrak{z}^\alpha$$

or

$$a_\beta{}^* = \frac{a_\beta}{1 - v^2/c^2} + \frac{v_\beta v_\alpha a^\alpha}{c^2(1 - v^2/c^2)^2}, \quad a^\alpha = a_\alpha$$

and, consequently, the following vector equality, which must obviously be satisfied generally in all the other forms of local inertial tetrads, as well as in special tetrads, in spatial points on curves of the family K, is true in each orthonormal inertial tetrad:

$$\mathbf{a}^* = \frac{d\mathbf{v}}{dt}\frac{1}{1 - v^2/c^2} + \frac{\mathbf{v}(\mathbf{v}\,d\mathbf{v}/dt)}{c^2}\frac{1}{(1 - v^2/c^2)^2}. \tag{1}$$

Formula (1) can be applied in two different Riemmanian spaces, as well as in the observer's reference system and in the proper global reference system in the same Minkowski space. It is obvious that in proper reference systems with $\mathbf{v} = 0$, $\mathbf{a}^* = \mathbf{a}$. The relationship (1) between the accelerations at points of the family K in Newtonian mechanics and in the special theory of relativity is, generally speaking, non-holonomic.

For a given family K of world lines, after the gravitational acceleration field $\mathbf{a}$ (or $\mathbf{g}$) in Newtonian mechanics has been theoretically determined with the support of experimental data and taking into account the velocity field, we can work out the gravitational acceleration field in the Special Theory of Relativity according to formula (1). After this, we can determine the gravitational acceleration field for other pseudo-Riemannian spaces described in terms of their metrics for the family K in the associated coordinates.

Under corresponding transformations of the spaces, in the general case the components of the metric tensor do not leave the form $ds^2 = g_{ij}d\xi^i d\xi^j$ invariant.

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